LIMIT AND CONTINUITY

非管理學院 (理,工,資電,地科學院)

Problems:

1. Give the $\epsilon - \delta$ definition of $\lim_{x \to a} f(x) = l$.

2. Use $\epsilon - \delta$ definition to prove the following :

$$if \quad \lim_{x \to a} f(x) = l, \quad \lim_{x \to a} g(x) = m$$

$$then \quad \lim_{x \to a} f(x) + g(x) = l + m$$

3. State the Sandwich Theorem. (i.e. Squeezing Theorem)

4. Give the definition of "f is continuous at a".

5. f(x)=$\begin{cases} x^2 + 2 & x < 1 \\ \frac{1}{x+1} & x > 1 \end{cases}$
   Write down (1) $\lim_{x \to 1^-} f(x)$ (2) $\lim_{x \to 1^+} f(x)$ (3) $\lim_{x \to 1} f(x)$

6. g(x)=$\begin{cases} x^3 + 1 & x < 0 \\ 3 & x = 0 \\ x + 2 & x > 0 \end{cases}$
   (i)Write down (1) $\lim_{x \to 0^-} g(x)$ (2) $\lim_{x \to 0^+} g(x)$ (3) $\lim_{x \to 0} g(x)$
   (ii)Is g continuous at 0 ? Why ?

7. h(x)=$\begin{cases} x^2 + 1 & x < -1 \\ 2 & x = \quad 1 \\ -2x & x > -1 \end{cases}$
   (i)Write down (1) $\lim_{x \to -1^-} h(x)$ (2) $\lim_{x \to -1^+} h(x)$ (3) $\lim_{x \to -1} h(x)$
   (ii)Is h continuous at -1 ? Why ?

8. f(x)=$\begin{cases} x & x \ is \ rational \\ -x & x \ is \ irrational \end{cases}$
   (i)Write down (1) $\lim_{x \to 0} f(x)$ (2) $\lim_{x \to 1} f(x)$
   (ii)Write down the points at which f is continuous.

9. Write down $\lim_{x\to 0} \sin \frac{1}{x}$, and justify your answer.
10. Write down $\lim_{x\to 0} x \sin \frac{1}{x}$, and justify your answer.
11. Give the definition of $\lim_{x\to a} f(x) = \infty$